UIMO
Unified International
Mathematics Olympiad

UNIFIED INTERNATIONAL MATHEMATICS OLYMPIAD

CLASS - 9
Question Paper Code : 4P104

KEY

1	2	3	4	5	6	7	8	9	10
A	B	D	C	A	D	A	A	C	A
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
C	B	C	D	B	B	B	B	A	A
21	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
D	C	A	C	C	D	B	B	D	C
31	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**
A,C,D	A,B,C	A,C	A,B,C	A,C,D	C	D	C	D	A
41	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**
C	C	C	B	D	A	C	D	B	A

SOLUTIONS

MATHEMATICS - 1 (MCQ)

01. (A) $\sqrt{4a^2+ab^2+16c^2+12a-24bc-16ca}$ =

$\sqrt{(2a)^2+(3b)^2+(-4c)^2+2(2a)(3b)}$ +

2(3b)(–4c)+2(–4c)(2a)

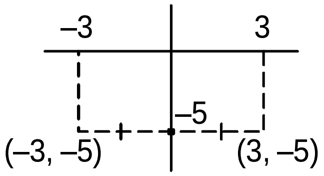
$=\sqrt{(2a+3b-4c)^2}$

02. (B) $x^2 + 2x + 1 - x^2 + 1 = 2x^2 + x - 2(x^2 + 3x + 2) + 20$

$2x + 2 = 2x^2 + x - 2x^2 - 6x - 4 + 20$

$7x = 16 - 2 \Rightarrow x = 2$

03. (D)



04. (C) ΔADH ≅ ΔJIH [∵ ASA congrueny]

∴ Area of ΔADH = area of ΔJIH

∴ Shaded area : Total area = 1 : 3 = $\frac{1}{3}$

05. (A) Given $\frac{4}{3}\pi r^3 = \frac{1}{3}\pi \times 6 \times 6 \times 24$ cm3

$r = 6$ cm

06. (D) $a^2 + 6ab + 9b^2 + b^2 + 2bc + c^2 + 4c^2 - 16c + 42 = 0$

$(a + 3b)^2 + (b + c)^2 + (2c - 4)^2 = 0$

Sum of three perfect squares is zero then each term must be zero

$\therefore$ a = – 3b, b = – c, 2c = 4

a = 6, b = – 2, c = 2

$\therefore$ a – b + c = 6 + 2 + 2 = 10

07. (A) $9^{\frac{1}{3}}, 11^{\frac{1}{4}}, 17^{\frac{1}{6}}$

$9^{\frac{4}{12}}, 11^{\frac{3}{12}}, 17^{\frac{2}{12}}$

$\sqrt[12]{9^4}, \sqrt[12]{11^3}, \sqrt[12]{17^2}$

$x = \sqrt[12]{6561}$ $y = \sqrt[12]{1331}$ $z = \sqrt[12]{289}$

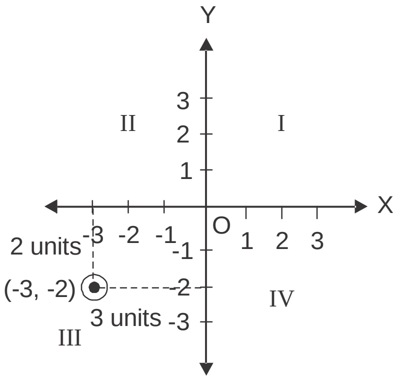
$\therefore x > y > z.$

08. (A) The perpendicular distance of a point from x-axis = 2 units.

The perpendicular distance of a point from y-axis = 3 units

Given, that the point lies in the III Quadrant

$\Rightarrow$ Both the coordinates of the point are negative.



$\therefore$ The required coordinates of the point are (–3, –2).

09. (C)

$$S = \frac{a+b+c}{2} = \frac{21m + 20m + 13m}{2} = \frac{54m}{2} = 27m$$

$$\Delta = \sqrt{S(s-a)(s-b)(s-c)}$$

$$= \sqrt{27 \times 6 \times 7 \times 14}\ m^2$$

$$= \sqrt{3 \times 9 \times 2 \times 3 \times 7 \times 2 \times 7}\ m^2$$

$= 3 \times 3 \times 2 \times 7\ m^2$

$= 126\ m^2$

10. (A) Given $2\cancel{\pi}r = 14\cancel{\pi}$ cm

$$r = \frac{14cm}{2} = 7cm$$

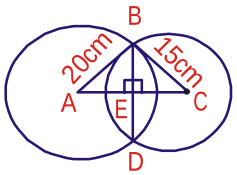
$\therefore$ Height = 2r = 14 cm

$$\text{Volume} = \pi r^2 h = \frac{22}{\cancel{7}} \times \cancel{7} \times 7 \times 14 cm^3$$

= 2156 cm3

$$= \frac{2156}{1000} \text{Litres} = 2.156 \text{ Litres}$$

11. (C) Given AB = 20 cm & BC = 15 cm



AC = 25cm and BD $\perp$ AC

In ΔABE, Let AE = $x$cm $\Rightarrow$ EC = $(25 - x)$ cm

$BE^2 = AB^2 - AE^2 = (20)^2 - x^2 = 400 - x^2$ $\rightarrow$ (1)

In ΔBCE, $BE^2 = BC^2 - EC^2 = (15)^2 - (25 - x)^2$

$= 225 - (625 - 50x + x^2)$

$= 225 - 625 + 50x - x^2$

$= 50x - x^2 - 400$ $\rightarrow$ (2)

But eq (1) = eq (2)

$$400 - \cancel{x^2} = 50x - \cancel{x^2} - 400$$

$400 + 400 = 50x$

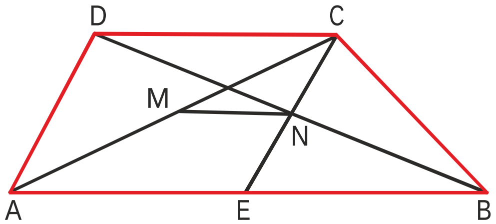
$$x = \frac{80\cancel{0}}{5\cancel{0}} = 16$$

$\therefore$ $BE2 = 400 - x^2 = 400 - 162 = 400 - 256 = 144$

$\therefore$ $BE = \sqrt{144}cm = 12cm$

$\therefore$ BD = 2BE = 2 × 12cm = 24 cm

12. (B)



Const: Join CN and extend upto E

Proof: $\Delta CND \cong \Delta ENB$

[$\because$ ASA Congruency]

$\therefore$ CN = NE $\Rightarrow$ 'N' is the mid point of CE & CD = BE

In ΔACE, MN = $\frac{1}{2}$ AE = $\frac{1}{2}$(AB – BE)

= $\frac{1}{2}$(AB – CD) = 3 cm

13. (C) $\frac{1}{x} = \frac{1}{9+4\sqrt{5}} \times \frac{9-4\sqrt{5}}{9-4\sqrt{5}} = \frac{9-4\sqrt{5}}{9^2-(4\sqrt{5})^2}$

= $9 - 4\sqrt{5}$

$$x + \frac{1}{x} = 9 + 4\sqrt{5} + 9 - 4\sqrt{5} = 18$$

Squaring on both sides

$$\left(x + \frac{1}{x}\right)^2 = 18^2$$

$$x^2 + 2 \times \cancel{x} \times \frac{1}{\cancel{x}} + \frac{1}{x^2} = 324$$

$$x^2 + \frac{1}{x^2} = 324 - 2 = 322$$

14. (D) Given $(x-1)^8 = a^8x^8 + a^7x^7 + a^6x^6 + \ldots + a^1x + a^0$

Given $(x-1)$ is a factor of $f(x) = (x-1)^8$

$\therefore$ f(1) = 0

$\therefore$ $f(1) = a^8(1)^8 + a^7(1)^7 + a^6(1)^6 + \ldots. + a^1(1)^1 + a^0 = 0$

$\therefore$ $a^8 + a^7 + a^6 + a^5 + a^4 + a^3 + a^2 + a^1 + a^0 = 0$

15. (B) From options If $x = 3$ then

$\sqrt{3+1} + \sqrt{6+3} = 2 + 3 = 5$ (or)

Given $\sqrt{x+1} + \sqrt{2x+3} = 5$

Squaring on both sides

$$\left(\sqrt{x+1} + \sqrt{2x+3}\right)^2 = 5^2$$

$$x + 1 + 2\sqrt{x+1}\sqrt{2x+3} + 2x + 3 = 25$$

$$2\left(\sqrt{x+1}\right)\left(\sqrt{2x+3}\right) = 25 - 4 - 3x$$

$$2\left(\sqrt{x+1}\sqrt{2x+3}\right) = 21 - 3x$$

Squaring on both sides

$4(x+1)(2x+3) = (21-3x)^2 = 441 - 126x + 9x^2$

$4(2x^2 + 5x + 3) = 441 - 126x + 9x^2$

$8x^2 + 20x + 12 - 9x^2 + 126x - 441 = 0$

$-x^2 + 146x - 429 = 0$

$x^2 - 146x + 429 = 0$

$x^2 - 143x - 3x + 429 = 0$

$x(x - 143) - 3(x - 143) = 0$

$\therefore$ $(x-3)(x-143) = 0$

$x = 3$ (or) $x = 143$ but $x = 143$ does n't satisfy the given question

$\therefore$ $x = 3$

16. (B) Let the height be 'x'

$$\therefore \text{Radius} = 1\frac{2}{3}x = \frac{5}{3}x$$

Given $2\pi rh$ = 4620 cm2

$$\Rightarrow 2 \times \frac{22}{7} \times \frac{5x}{3} \times x = 4620 \text{ cm}^2$$

$$x^2 = \cancel{4620}^{\cancel{210}^{\cancel{105}21}} \text{cm}^2 \times \frac{1}{\cancel{2}} \times \frac{7}{\cancel{22}_1} \times \frac{3}{\cancel{5}_1}$$

$x^2 = (21\text{cm})^2$

$\therefore x = 21$ cm

$$\therefore \text{Radius} = \frac{5}{3}x = \frac{5 \times \cancel{21}^7 \text{cm}}{\cancel{3}_1} = 35\text{cm}$$

Total surface area = $2\pi r$ (h + r)

$$= 2 \times \frac{22}{\cancel{7}_1} \times \cancel{35}^5 \text{cm}(21 + 35)\text{cm}$$

= 220 cm × 56cm = 12320 cm^2

17. (B) (a + b + c)2 – (a – b – c)2 = (a2 + b2 + c2 + 2ab + 2bc + 2ca) – (a2 + b2 + c2 – 2ab + 2bc – 2ca)

= a2 + b2 + c2 + 2ab + 2bc + 2ca – a2 – b2 – c2 + 2ab – 2bc + 2ca

= 4ab + 4ca

= 4a (b + c)

18. (B) LHS $\sqrt{\sqrt{3}-\sqrt{4-\sqrt{5}-\sqrt{12+5-2\sqrt{12}\times\sqrt{5}}}}$

$$\sqrt{\sqrt{3}-\sqrt{4-\sqrt{5}-\sqrt{\left(\sqrt{12}-\sqrt{5}\right)^2}}}$$

$$= \sqrt{\sqrt{3}-\sqrt{4-\sqrt{5}-\left(\sqrt{12}-\sqrt{5}\right)}}$$

$$= \sqrt{\sqrt{3}-\sqrt{4-\sqrt{5}-\sqrt{12}+\sqrt{5}}}$$

$$= \sqrt{\sqrt{3}-\sqrt{4-\sqrt{12}}}$$

$$= \sqrt{\sqrt{3}-\sqrt{\left(\sqrt{3}\right)^2+1^2-2\sqrt{3}}}$$

$$= \sqrt{\sqrt{3}-\sqrt{\left(\sqrt{3}-1\right)^2}}$$

$$= \sqrt{\sqrt{3}-\sqrt{3}+1}$$

$= \sqrt{1} = 1$

19. (A) Given $x^2 + x + 1 = 0$

$$\frac{x^2}{x}+\frac{x}{x}+\frac{1}{x}=0$$

$$x+1+\frac{1}{x}=0$$

$$x+\frac{1}{x}=-1$$

cubing on both sides

$$x^3+\frac{1}{x^3}+3x\times\frac{1}{x}\left(x+\frac{1}{x}\right)=-1$$

$$x^3+\frac{1}{x^3}+3(-1)=-1$$

$$x^3+\frac{1}{x^3}=-1+3$$

$$x^3+\frac{1}{x^3}=2$$

cubing in both sides

$$\left(x^3+\frac{1}{x^3}\right)^3=8$$

(OR) Given $x^2 + x + 1 = 0$

$(x – 1)(x^2 + x + 1) = 0\,(x – 1)$

$x^3 – 1^3 = 0 \Rightarrow x^3 = 1$

$$\therefore \left(x^3+\frac{1}{x^3}\right)^3=\left(1+\frac{1}{1}\right)^3$$

$= 2^3 = 8$

20. (A) Given $\angle A + \angle C = 140^o$

and $\angle A : \angle C = 1 : 3$

$$\Rightarrow \angle A = 140^o \times \frac{1}{4} = 35^o$$

$$\text{and } \angle C = 140^o \times \frac{3}{4} = 35^o \times 3 = 105^o$$

In the quadrilateral

$\angle A + \angle B + \angle C + \angle D = 360^o$

$\Rightarrow \angle B + \angle D = 360^o – (\angle A + \angle C)$

$= 360^o – 140^o$

$\therefore$ $\angle B + \angle D = 220^o$

Given that $\angle B : \angle D = 5 : 6$,

$$\angle B = 220^o \times \frac{5}{11} = 20^o \times 5 = 100^o$$

$$\text{and } \angle D = 220^o \times \frac{6}{11} = 20 \times 6 = 120^o$$

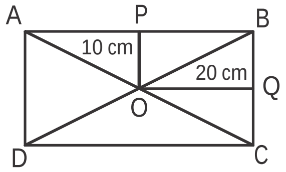
$\therefore$ The required angles are $\angle A = 35^o$, $\angle B = 100^o$, $\angle C = 105^o$ and $\angle D = 120^o$.

21. (D) BC = AD = 2PO = 20 cm

AB = DC = 2 × OQ = 40 cm

Perimeter of rectangle = 2(AB +BC)

= 120 cm



22. (C) Given $\left(\frac{d_1}{2}\right)^2 + \left(\frac{d_2}{2}\right)^2 = \left(\frac{34}{4}\text{ cm}\right)^2$

$$\frac{d_1{}^2}{4} + \frac{d_2{}^2}{4} = \frac{289}{4}\text{ cm}^2$$

$\therefore\ d_1^2 + d_2^2 = 289$

Given $d^1 + d^2 = 23$ cm

squaring on both sides

$d_1^2 + d_2^2 + 2d_1 d_2 = 529$

$289 + 2d^1\, d^2 = 529$

$2d^1\, d^2 = 240$

$$\frac{2d_1 d_2}{4} = \frac{240}{4}\text{ cm}^2$$

Area of rhombus = $\frac{d_1 d_2}{2}$ = 60 cm^2

23. (A) Volumes ratio of sphere, cone & cylinder

$= \frac{4}{3}\pi r^3 : \frac{1}{3}\pi r^2 h : \pi r^2 h$

$= \frac{4}{3} : \frac{2}{3} : 2$

$= \frac{2}{3} : \frac{1}{3} : 1$

= 2 : 1 : 3

24. (C) Let the original radius be r cm

$\therefore$ Original surface area = $4\pi r^2$

New radius (R) = r + 100% r = r + r = 2r

New are surface = $4\pi R^2 = 4\pi(2r)^2$

= $4(4\pi R^2)$

Increased area = $4(4\pi R^2) - 4\pi r^2$

= $3(4\pi r^2)$

Increased area percentage

$= \frac{3(4\pi r^2)}{4\pi r^2} \times 100$ = 300 %

25. (C) Given $x + \frac{9}{x} = 6 \Rightarrow \frac{x^2 + 9}{x} = 6$

$\Rightarrow\ x^2 - 6x + 9 = 0$

$\Rightarrow\ x^2 - 3x - 3x + 9 = 0$

$\Rightarrow\ x = 3$

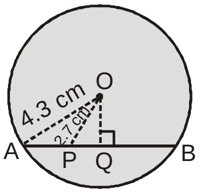
$\therefore\ x^2 + \frac{9}{x^2} = 9 + \frac{9}{9} = 9 + 1 = 10$

26. (D) $\Delta AOX \cong \Delta COY$ [$\because$ ASA congruency]

$\therefore$ OX = OY [$\because$ CPCT]

$\therefore$ OX – OY = 0

27. (B) As shown in the figure, OP = 2.7 cm and OA = 4.3 cm. Draw a perpendicular OQ to the chord AB. Clearly, AQ = QB. Since P divides AB in the ratio 7 : 10, let AP be 7x and PB be 10x.



Also, PQ = AQ – AP = $\frac{AB}{2}$ – AP

$= \frac{17x}{2} - 7x = 1.5x$

By Pythagoras' theorem, we have, AQ^2 + OQ^2 = AO^2.

Also, OP^2 = PQ^2 + OQ^2.

$\Rightarrow$ AQ^2 – PQ^2 = AO^2 – OP^2

$\Rightarrow$ $(8.5x)^2 - (1.5x)^2 = (4.3)^2 - (2.7)^2$

$\Rightarrow$ $(10x)\,(7x) = 7(1.6)$

$\Rightarrow$ $x^2 = 0.16$ or $x = 0.4$ cm.

$\therefore$ AB = $17x = 17 \times 0.4 = 6.8$ cm

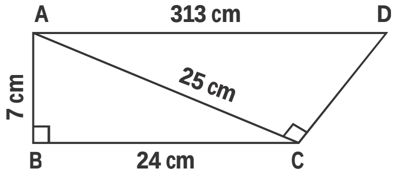
28. (B) $\angle RQP = 30^\circ + 225^\circ = 55^\circ$

$\angle RQP = +\ x = 180^\circ$

$55^\circ + x = 180^\circ$

$x = 180^\circ - 55^\circ = 125^\circ$

29. (D)



In ΔABC, $\angle B = 90^\circ$

AC^2 = AB^2 + BC^2

AB = 7 cm

In ΔACD, $\angle ACD = 90^\circ$

AD^2 = AC^2 + CD^2

313^2 – 25^2 = CD^2

97,969 – 625 = CD^2

CD = $\sqrt{97344}$

CD = 312 cm

Area of quad ABCD = Area of ABC + Area of ACD

$$= \frac{1}{2} \times AB \times BC + \frac{1}{2} \times AC \times CD$$

$$= \frac{1}{2} \times 7\ \text{cm} \times 24\ \text{cm} + \frac{1}{2} \times 25\ \text{cm} \times 312\ \text{cm}$$

= 84 cm^2 + 3900 cm^2

= 3984 cm^2

30. (C) ARPQ is a parallelogram

$\therefore$ AR = PQ & PR = AQ

$$\therefore AR = PQ = \frac{AB}{2} = \frac{30cm}{2};$$

$$PR = AQ = \frac{AC}{2} = \frac{21}{2} cm$$

$$\therefore AR + RP + PQ + QA$$
$$= \frac{30cm}{2} + \frac{30cm}{2} + \frac{21cm}{2} + \frac{21cm}{2}$$
$$= 51cm$$

MATHEMATICS - 2 (MAQ)

31. (A,C,D) Given $x = \frac{1}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}} = 2 + \sqrt{3}$

$x - 2 = \sqrt{3}$

Squaring on both sides

$x^2 - 4x + 4 = 3$

$x^2 - 4x + 1 = 0$

$$x^2 - 4x + 1)x^4 - x^3 + 6x^2 - 17x + 16(x^2 + 3x + 5$$
$$x^4 - 4x^3 + x^2$$
$$(-)\ \ (+)\ \ (-)$$
$$3x^3 - 7x^2 - 17x + 16$$
$$3x^3 - 12x^2 + 3x$$
$$(-)\ \ (+)\ \ (-)$$
$$5x^2 - 20x + 16$$
$$5x^2 - 20x + 5$$
$$(-)\ \ (+)\ \ (-)$$
$$(11)$$

$$x^2 - 4x + 1)x^3 - 2x^2 - 7x + 2(x + 2$$
$$x^3 - 4x^2 + x$$
$$(-)\ \ (+)\ \ (-)$$
$$2x^2 - 8x + 2$$
$$2x^2 - 8x + 2$$
$$(0)$$

$$x^2 - 4x + 1 \left| \begin{array}{c} x^4 - x^3 - 6x^2 - 17x + 5 \\ x^4 - 4x^3 + x^2 \\ (-)\ \ (+)\ \ (-) \\ \hline 3x^3 - 7x^2 - 17x + 5 \\ 3x^3 - 12x^2 + 3x \\ (-)\ \ (+)\ \ (-) \\ \hline 5x^2 - 20x + 5 \\ 5x^2 - 20x + 5 \\ \hline (0) \end{array} \right| x^2 + 3x + 5$$

32. (A,B,C) Given p(x) = $x^{2024} - y^{2024}$

p(y) = $(y)^{2024} - y^{2024}$

$= y^{\cancel{2024}} - y^{\cancel{2024}}$

p(y) = 0 ($x - y$) is a factor of p(x)

p($-y$) = $(-y)^{2024} - y^{2024}$

$= y^{\cancel{2024}} - y^{\cancel{2024}}$

p($-y$) = 0 ($x + y$) is a factor of p(x)

($x + y$) and ($x - y$) are factors of p(x)

($x^2 - y^2$) is also a factor of p(x)

33. (A,C) Given p(x) = x^3 q^2 – x^3 pt + $4x^2$ pt – $4x^2$ q^2 + $3xq^2$ – $3x$ pt

$\therefore$ p(1) = q^2 – pt + 4pt – $4q^2$ + $3q^2$ – 3pt = 0

(x–1) is a factor of p(x)

p(3) = $27q^2$ – 27pt + 36pt – $36q^2$ + $9q^2$ – 9pt = 0

$\therefore$ (x–3) is a factor of p(x)

34. (A,B,C)

Except option (D) all the lines don't pass through origin.

35. (A,C,D)

Square and rhombus are also parallelograms

REASONING

36. (C) Difference between the digits of 37 is 7 – 3 = 4. In the others, this rule is not satisfied.

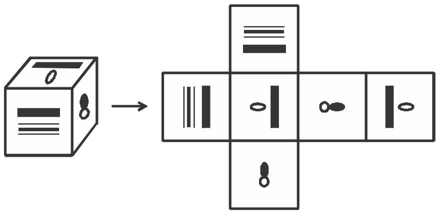
37. (D)

$$\begin{array}{l|l} 7^2-5^2=24 & 22^2-20^2=84 \\ 5^2-3^2=16 & 11^2-9^2=40 \\ 6^2-4^2=20 & 9^2-7^2=32 \\ 3^2-1^2=8 & \boxed{10}^2-8^2=36 \end{array}$$

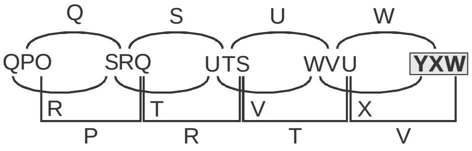
38. (C) GEYAAWT – GETAWAY means 'a quick departure'.

39. (D)

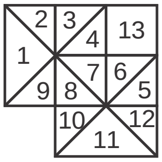
40. (A)



41. (C)



42. (C)



Small triangles → ⑫

2 + 3, 4 + 7, 7 + 6, 5 + 12, 10 + 8, 9 + 8 → ⑥

1 + 2 + 3, 8 + 10 + 11, 5 + 11 + 12, 1 + 8 + 9 → ④

2 + 3 + 4 + 7, 4 + 7 + 8 + 9, 6 + 7 + 8 + 10, 7 + 6 + 5 + 12 → ④

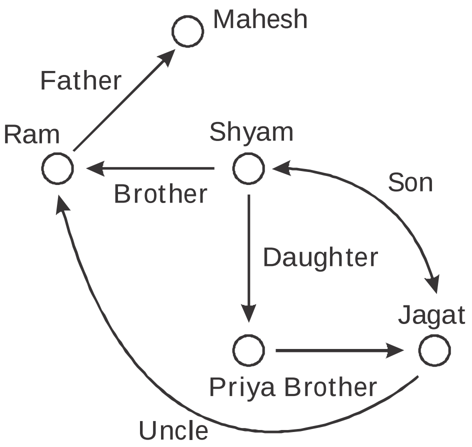
2 + 3 + 4 + 7 + 13 + 6 + 5 + 12 → ①

∴ Total number of triangles

= 12 + 6 + 4 + 4 + 1 = 27

Hence, there are 27 triangles.

43. (C) Jagat is the brother of Priya and Priya is the daughter of Shyam. Therefore Shyam is the father of Jagat. Ram is the brother of Shyam. Therefore, Ram is the uncle of Jagat.



44. (B) Shapes in the right diagonal interchange and the two shapes in the left top corner and right bottom corner interchange places and top corner gets a new shape.

45. (D) Code for white fill is G, and code for hexagon is R.

Hence, the code for ⬡ is **RG**.

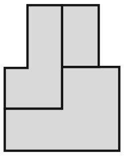
CRITICAL THINKING

46. (A) The Reason (R) does provide a potential explanation for the Assertion (A), as the burning of organic material (stubble) can result in plumes that contribute significantly to air pollution levels.

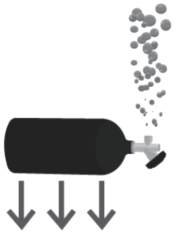
Given this analysis, both Assertion (A) and Reason (R) are true, and Reason (R) provides a correct explanation of Assertion (A).

Answer : (A) If both (A) and (R) are true and (R) is the correct explanation of (A).

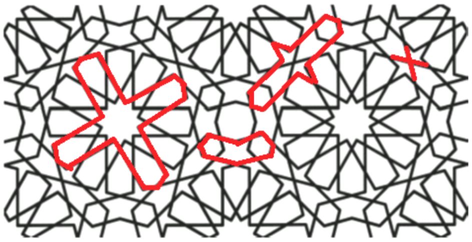
47. (C)



48. (D) As air escapes the available space is quickly replaced with water, so the tank' density becomes the same as that of the water and with the added weight and density of the tank itself continues to sink.



49. (B) (i), (iii), (v), (vi)



50. (A) Let's analyze each statement considering only one person is telling the truth.

1. If Aarav is telling the truth (Aarav didn't break the vase):

- Aarav is telling the truth.
- Bhavana is lying (Aarav didn't break the vase).
- Charan is lying (Bhavana did break the vase).
- Diya is lying (Charan is telling the truth).
- This means Bhavana must have broken the vase, which contradicts Aarav 's statement.

2. If Bhavana is telling the truth (Aarav broke the vase):

- Bhavana is telling the truth.
- Aarav is lying (Aarav did break the vase).
- Charan is lying (Bhavana did break the vase).
- Diya is lying (Charan is telling the truth).
- This means Aarav must have broken the vase, which doesn't lead to any contradictions.

3. If Charan is telling the truth (Bhavana didn't break the vase):

- Charan is telling the truth.
- Aarav is lying (Aarav did break the vase).
- Bhavana is lying (Aarav didn't break the vase).
- Diya is lying (Charan is lying).
- This contradicts the assumption that only one person is telling the truth.

4. If Diana is telling the truth (Charan is lying):

- Diya is telling the truth.
- Aarav is lying (Aarav did break the vase).
- Bhavana is lying (Aarav didn't break the vase).
- Charan is lying (Bhavana did break the vase).
- This means Bhavana must have broken the vase, which contradicts the assumption that Charan is lying about Bhavana.

Therefore, the only consistent scenario is when Bhavana is telling the truth, which means Aarav broke the vase.